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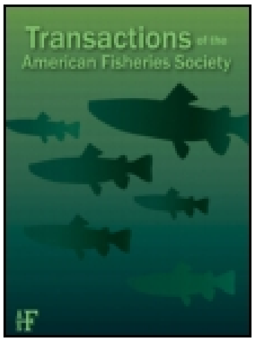
Clay Porch
National Oceanic and Atmospheric Administration



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Simulating the Dynamic Trends of Fisheries Regulated by Small Daily Bag Limits

CLAY E. PORCH AND WILLIAM W. FOX, JR.[1]

Cooperative Institute for Marine and Atmospheric Studies
Rosenstiel School of Marine and Atmospheric Science, University of Miami
4600 Rickenbacker Causeway, Miami, Florida 33149, USA

Abstract.—We developed an algorithm for simulating the effects of potential bag limits. The algorithm uses statistical distributions to model the frequency distribution of catch per trip. The moment equations of the distribution are keyed to stock abundance and the coefficient of fishing mortality rate, which allow the model to reflect the dynamics of the fishery. The catch with the bag limit is calculated by censoring the simulated distribution. We establish the conditions under which compounded Poisson distributions, such as the negative binomial, can be expected to provide adequate models for the catch distributions of recreational fisheries. We also discuss the assumptions that underlie the method of censoring catch-per-trip distributions, and, where appropriate, suggest alternative models. An example is developed in which we use the negative binomial distribution to model catch per trip and the familiar exponential model to simulate the change in stock abundance due to fishing mortality.

A bag limit is a type of allocated catch quota that restricts the number of animals a fisher, or group of fishers, can keep per unit time. Hence, a bag limit reduces the catch made by the more skilled, or lucky, fishers. The usual purposes of bag limits are to reduce fishing mortality and to allocate the resource more equitably. Daily bag limits have proven especially popular among managers of recreational fisheries, because the limits can be used to prolong the open season under a catch quota and because the choice of the most appropriate size for the bag limit does not (necessarily) require estimates of the total catch or total effort.

Prudent implementation of a bag limit program requires answers to two basic questions: (1) What size bag limit will most likely achieve the prescribed immediate (short-term) reduction in fishing mortality? and (2) How should the bag limit be adjusted over the long term to accommodate changes in stock abundance or fishing practices of the fishers? Previous studies, for example, Eldridge and Powers (1983) and Bannerot (1987), addressed the first question with the traditional censoring procedure in which catches that are observed to exceed the bag limit are counted as equal to the bag limit. Both studies pointed out some of the inherent limitations of the traditional approach, but with the exception of an adjustment for release mortality by Bannerot (1987), did not propose alternative methods. Argue et al. (1983) examined the second question to some extent, but did not allow for changes in fishing success with changing stock abundance. Ideally, one would like to know how both fishing success and stock abundance will change when a given bag limit, or sequence of several bag limits, is implemented over an extended period of time. Insight into this problem would be especially helpful to managers who wish to implement a regulatory program over several years.

We present an algorithm for simulating the effects of potential bag limits on a fishery. The algorithm links models for the frequency distribution of catch per trip, the effect of a bag limit on catch per trip, and the effect of fishing on stock abundance. We discuss these models in detail and develop an example with the negative binomial distribution and a simple exponential model of stock abundance.

Modeling the Frequency Distribution of Catch Per Trip

Greenwood and Yule (1920) showed that the frequency of multiple accidents sustained by individuals working in a factory can be modeled by some form of compound Poisson distribution. Thompson (1976) and others have since suggested that the same may be true for frequency distributions of catch per fishing trip. In this section we show that, under certain conditions, compound Poisson models are indeed appropriate for catch distributions. Our derivation partially follows the

[1] Present address: National Marine Fisheries Service, 1335 East–West Highway, Silver Spring, Maryland 20910, USA.

approach of Greenwood and Yule (1920), but our assumptions are sufficiently different to warrant some discussion here. Other models, specifically for hook-and-line angling, have been derived by Deriso and Parma (1987).

The observed frequency distribution of catch per trip is a composite of the catches of S total trips; a trip may be defined either in terms of a single fisher (fisher-trip) or in terms of a group of fishers (party-trip or vessel-trip). We modeled fisher-trips as a series of n fishing instants; a fishing instant is defined as a unit of time sufficient in duration to catch a single fish. The probability (p) of catching a fish during the ith instant of the jth fisher-trip, p_{ij}, is considered constant throughout the duration of that instant, but the p_{ij} values can vary among instants and fisher-trips.

Under these assumptions, the fishing process may be addressed as a series of n independent Bernoulli trials, each with a unique probability p_{ij} for success:

$$(p_{1j} + q_{1j})(p_{2j} + q_{2j})\ldots(p_{nj} + q_{nj}) = 1; \quad (1)$$

$q_{ij} = 1 - p_{ij}$. The probability of catching x fish over the course of the jth fisher-trip is given by the summation of like terms in the expansion of equation (1), commonly referred to as the generalized binomial distribution. Total catch (C) is distributed as the amalgam of the S separate generalized binomial distributions. The expectation is $E[C] = \Sigma_i \Sigma_j p_{ij}$.

Strictly speaking, equation (1) applies only to *singular-capture* fisheries in which the chance of landing more than one fish simultaneously is remote (e.g., angling or spearfishing). However, because this condition typifies most recreational fisheries, the model should find broad application. Multiple-capture fisheries (e.g., trawling) will probably require a different model (see, for example, Pella and Psaropulos 1975 or Mangel and Beder 1985).

Recording the instant-by-instant fishing success for every fisher-trip is at best impractical. Therefore, one must take several simplifying assumptions regarding the p_{ij} values:

(1) p_{ij} is small for all i (instants) and j (fisher-trips), i.e., the probability of catching a fish during any given instant on any given fisher-trip is small;
(2) the number of instants (maximum possible catch) in the jth trip, n_j, is large; and
(3) the expected catch for the jth trip, $\lambda_j = \Sigma_i p_{ij}$, varies among the S total fisher-trips as a random variable (i.e., λ_j is a real-valued function defined on a probability space).

When assumptions (1) and (2) are satisfied, the generalized binomial distribution tends to the Poisson distribution (Patil and Joshi 1968):

$$P[x \mid \lambda_j] = \frac{\lambda_j^x}{x!} e^{-\lambda_j}; \quad (2)$$

$P[x \mid \lambda_j]$ denotes the probability of catching exactly x fish on the jth fisher-trip. Notice that we do not need to assume that the probability of catching a fish is constant over all instants, only that it is small. Of course, in practice, the p_{ij} values will not always be small, so assumption (1) is not exactly satisfied. Nevertheless, if the maximum possible catch (n_j) is large, but the expected catch (λ_j) is moderate, the Poisson approximation will be fairly robust (Feller 1968).

Assumption (3) (that λ_j is a random variable) permits the single-fisher Poisson model to be generalized over all fisher-trips by allowing λ_j a probability distribution of its own—$f[\lambda_j]$. Such generalizations are commonly termed "compound Poisson" distributions, and are obtained by integrating the product of $f[\lambda_j]$ and equation (2) over the range of λ_j values. Compounding the Poisson density with the flexible gamma distribution, for example, results in the familiar negative binomial distribution (Greenwood and Yule 1920):

$$P[x] = \frac{(K + x - 1)!}{x!(K - 1)!}\left(\frac{m}{m + K}\right)^x\left(1 + \frac{m}{K}\right)^{-K}; \quad (3)$$

$P[x]$ is the probability of catching x fish on an average fisher-trip; m is the overall expected catch per trip ($m = \Sigma\, \lambda_j/S$), and K is a measure of heterogeneity about the mean ($K = m^2/[\sigma^2 - m]$; σ^2 is the variance in catch among trips).

So far we have dealt only with the theoretical catch-per-fisher-trip (c/f) distribution. However, if the catches of fishers within parties are independent, then the Poisson model also applies to the probability of catching a total of x fish on the jth party-trip. Realistically, the catches of fishers in a given party will seldom be completely independent, but unless the technologies being used differ greatly within parties, violations of independence would not seem to be of much consequence. The mathematical problem becomes much more difficult without the assumption of quasi-independence; it involves conditional probabilities, which can lead to complicated integrals without known solutions (Porch 1988).

Generalizing the single-party Poisson model over all party-trips is, in principle, the same as generalizing the single-fisher Poisson model over all fisher-trips, except that the compounding function $f[\lambda_i]$ would include variations in the number of fishers per party. It is also possible to generalize over subgroupings of party-trips, distinguished, perhaps, by the number of fishers per party or by some qualitative trait (e.g., charter versus private).

In summary, we have confirmed, from a theoretical standpoint, that the catch distributions of most recreational fisheries ought to follow closely some form of compound Poisson distribution. The negative binomial distribution seems promising when the distribution of expected catches among fishers or parties has a unimodal or decaying exponential-type pattern such that the gamma distribution would provide a reasonable fit. Empirical evidence supporting the negative binomial distribution as a model for c/f distributions has been reported by Bannerot and Austin (1983) and Small and Downham (1985). Examples of successful fits of the negative binomial distribution to catch-per-vessel-day distributions for spotted seatrout *Cynoscion nebulosus* in Florida Bay and king mackerel *Scomberomorus cavalla* off eastern Florida are shown in Figure 1.

Note that simple compounding functions, such as the two-parameter gamma, may not be sufficiently flexible to include the effects of spatial or temporal changes in the abundance or catchability of the stock. Hence, even if the Poisson model is entirely appropriate for individual trips, its generalization may not provide a good fit to data collected across a broad area or a long time span. Mathematically, this is essentially the same problem as that in ecological "clump" models in which the compounding function changes shape due to animal movements (Pielou 1977). Therefore, those wishing to test any model of catch per trip should keep in mind that it applies only to subsets of the data in which the conditions affecting $f[\lambda_i]$ were relatively homogenous. As we will show in the next section, the dynamic character of the catch-per-trip distribution is a critical part of any analysis of bag limits so that models for data aggregated over long time spans are not particularly useful in any case.

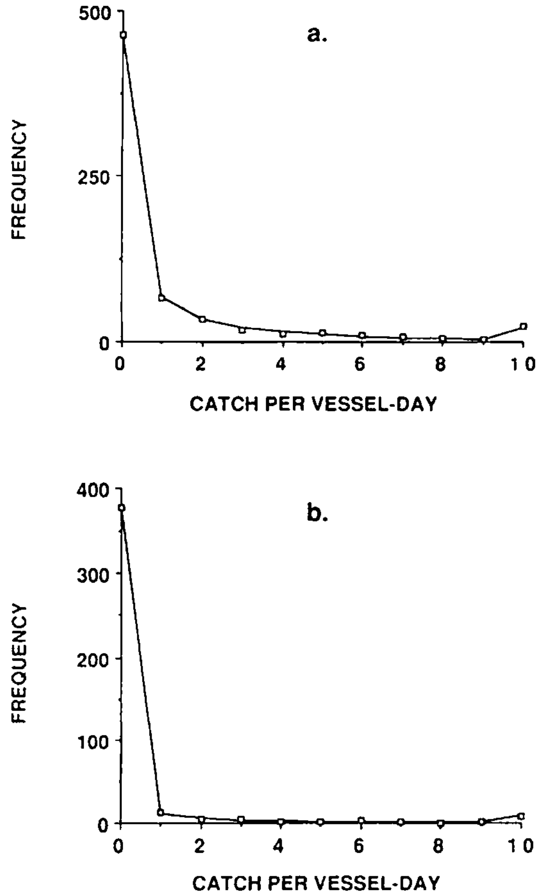

FIGURE 1.—Negative binomial fits to catch-per-vessel-day distributions for (a) recreational catch of spotted seatrout in Florida Bay during summer 1980 and (b) charter boat catch of king mackerel off east Florida during 1982.

Modeling the Effect of a Bag Limit on Catch

The total catch with a potential bag limit, ${}^bC$ (b denotes the size of the bag limit), is often estimated by censoring observed catch-per-fisher-trip distributions at the bag limit, that is, daily catches larger than the bag limit are counted as equal to the bag limit:

$$ {}^bC = S\left(\sum_{x<b} xP[x] + b\sum_{x\geq b} P[x]\right); \tag{4}$$

$P[x]$ is the proportion of the S total fisher-trips that caught x fish. This approach is reasonable, provided these assumptions are met:

(4) the shape of the underlying c/f distribution does not vary (but see below);
(5) when regulated by a bag limit, individual fishers cease fishing as soon as the bag limit is reached, but do not otherwise alter their behavior;
(6) The bag-limit-regulated fishery is not affected by other interests operating on the fishery; and
(7) the fishery targets and catches a single species.

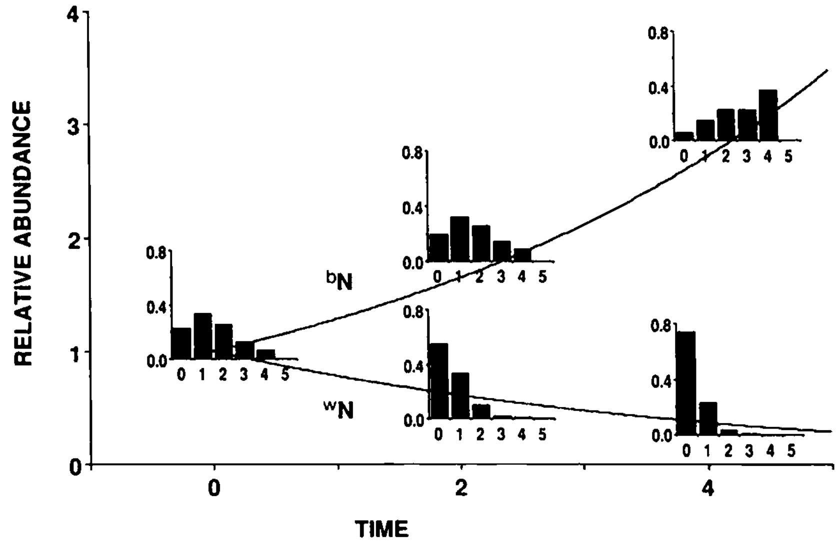

FIGURE 2.—Temporal divergence between stock abundance with the bag limit (^bN) and stock abundance without the bag limit (^wN), with corresponding divergence between the shapes of the censored catch-per-trip distributions. The vertical and horizontal axes of the major graph are relative abundance (the abundance at time t divided by the initial stock abundance, N_t/N_0) and time in arbitrary units (for example, 1 year). The vertical axes of the inset graphs are relative frequency (probability) and the horizontal axes are catch per trip. The specific pattern shown was generated by assuming that the rate of change in stock abundance is linearly related to the current stock abundance: $dN_t/dt = (G - F)N_t$; F (=1.0) is the instantaneous fishing mortality rate coefficient and G (=0.75) is the intrinsic growth rate coefficient. The bag limit (4 fish/d) was assumed to reduce the fishing mortality rate to roughly one-half of this magnitude without the bag limit. Catch per trip was assumed to be Poisson distributed with parameter λ (the overall mean catch per trip).

The individual assumptions can be relaxed to varying degrees, yielding a less unwieldy set of restrictions (see the discussion section). Assumption (4), however, is particularly difficult because it implies that catch per unit effort is independent of stock abundance. In reality, if the bag limit is successful in reducing fishing mortality, and all other things (initial stock abundance, fisher skill and effort, fish behavior, etc.) are equal, the mean stock abundance with the bag limit will be larger than it would have been without the bag limit. Hence, if catch per unit effort is density-dependent, the shape of the c/f distribution, $P[x]$, must differ between the two cases (Figure 2).

Strictly speaking then, equation (4) is exact only in the instantaneous sense:

$$d^bC_t = S_t\left(\sum_{x \le b} xP[x \mid t] + b \sum_{x > b} P[x \mid t]\right) dt; \tag{5}$$

$P[x \mid t]$ and S_t are the time-dependent parameters of the c/f distribution and d^bC_t is the catch landed during the infinitesimal time interval $[t, t + dt]$. Thus, in order to compute the catch with the bag limit that accumulates during a finite interval, it is necessary to integrate equation (5):

$$^bC = \int_{t_{start}}^{t_{end}} S_t\left(\sum_{x \le b} xP[x \mid t] + b \sum_{x > b} P[x \mid t]\right) dt. \tag{6}$$

Of course, to evaluate this integral one must define the c/f distribution ($P[x \mid t]$ and S_t) as a function of stock abundance (or other variables), and stock abundance as a function of time.

The c/f Distribution as a Function of Stock Abundance

Any model of the c/f distribution can be expressed in terms of its moments (mean, variance, etc.) and the total number of trips (S). Accordingly, the task of ascribing a dynamic character to the c/f distribution reduces to that of choosing the appropriate functional forms for its moments and for S.

The mean catch per trip during a given time period is simply the total catch divided by the total number of trips, C/S, and so can easily be obtained from any appropriate simulation routine. Most simulation models, however, do not define the variance and higher-order moments explicitly. In such cases, the only alternative is to express those moments as functions of variables that are defined by the model (e.g., C).

Taylor (1961) and Taylor et al. (1978) presented considerable empirical evidence, and some conceptual justification, for a simple power law relating spatial variance (σ^2) to mean population density (μ):

$$\sigma^2 = \alpha\mu^{\beta}. \quad (7)$$

Several authors have suggested that a similar law might apply to the relationship between the fishing mortality rate and stock abundance (Fox 1974; MacCall 1976; Peterman and Steer 1981). If this is true, Taylor's empirical law should also hold for the means and variances of catch per trip. Small and Downham (1985) reported that the means and variances of the daily catches of brown trout *Salmo trutta* and Atlantic salmon *Salmo salar* in British waters do in fact follow Taylor's law. Catch-per-vessel-trip data from the two fisheries we have examined, spotted seatrout in Florida Bay and king mackerel in the southeastern USA, also support this hypothesis (Figure 3).

Higher-order moments will be needed if the c/f distribution has more than three parameters (including S), but we suspect that similar empirical laws could be established for these as well.

The total number of trips (S) can vary in time for several reasons: inclement weather, changes in stock abundance, timing of alternate activities, or even as a response to the bag limit. Because this issue is rather involved, we defer all related discourse to the discussion section. The subscript notation, however, is retained to preserve the generality of subsequent developments.

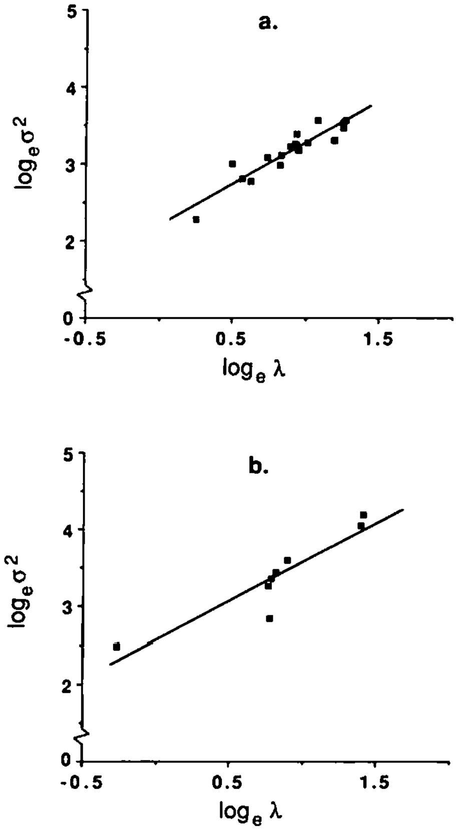

FIGURE 3.—Regressions of $\log_e(\sigma^2)$ against $\log_e(\lambda)$ from (a) quarterly summaries of the daily recreational landings of spotted seatrout in Florida Bay for the years 1980–1984 ($\sigma^2 = 9.14\lambda^{1.08}$; $r = 0.93$) and (b) annual summaries of the daily charter boat landings of king mackerel in east Florida and the Gulf of Mexico for the years 1982–1985 ($\sigma^2 = 13.51\lambda^{1.01}$; $r = 0.92$).

Change in Stock Abundance with the Bag Limit

Any continuous, time-dependent model of stock abundance can be reduced to the simple form

$$\frac{dN_t}{dt} = G_t N_t - \frac{dC_t}{dt}; \quad (8)$$

G_t is the difference between the intrinsic growth and death rates (per unit time) for the stock and dC_t/dt is the instantaneous catch rate (per unit time), neither of which is necessarily constant or independent of stock abundance N_t. As discussed previously, the instantaneous catch with the bag limit, d^bC_t, can be written as a function of the probability of catching x fish given a certain level of abundance, $P[x \mid N_t]$. In particular, if assumptions (5), (6), and (7) hold, we may write equation (8) as

$$\frac{dN_t}{dt} = G_t N_t - S_t \cdot \left(\sum_{x \le b} xP[x \mid N_t] + b \sum_{x > b} P[x \mid N_t] \right). \quad (9)$$

In practice, exact solutions to equations (8) or (9) will not be possible unless $P[x]$ is an unrealistically simple function of N_t. Hence, one must turn to some numerical recipe for an approximate solution. The algorithm below is one such recipe.

Numerical Algorithm

The solution to equation (8) for the time interval $[0, T]$ can be written

$$N_T = N_0 e^{\int_0^T (G_t - {}^bF_t)\, dt}. \quad (10)$$

The coefficient ${}^bF_t$, which represents the instantaneous fishing mortality rate with the bag limit (per unit time), is defined as

$${}^bF_t \equiv \frac{1}{N_t}\frac{d^bC_t}{dt},$$

but can also be written as a fraction of the fishing mortality rate without the bag limit by substituting in the identity $d^wC_t \equiv {}^wF_tN_t\,dt$ (w = without the bag limit):

$${}^bF_t = \frac{d^bC_t}{d^wC_t}\,{}^wF_t.$$

These equations suggest the approximating recursion

$$N_1 = N_{\text{start}}$$

$$N_{k+1} \approx N_k e^{(G_k - {}^bF_k)h_k}, \quad (11)$$

in which

$${}^bF_k \approx \frac{{}^bC_k}{{}^wC_k}\,{}^wF_k. \quad (12)$$

The parameter h_k is the duration of the kth time step in terms of the units used to define the rate coefficients (G_k and ${}^bF_k$); for example, if G_k and ${}^bF_k$ were expressed as annual rates, then h_k would be written as some fraction of a year. The rate coefficients G_k and ${}^bF_k$ are held constant throughout the duration of the kth time step, but are allowed to vary from step to step as functions of the conditions at the start of the kth step. The terms ${}^bC_k$ and ${}^wC_k$ are the catches made with and without the bag limit during the kth step, given an initial abundance N_k (more on this later). The recursion allows us to compute equation (10) numerically by piecing together a series of exponential approximations. Thus, for the time interval [0, T], divided into τ number of steps, the approximating formula is

$$N_T \approx N_1 e^{\sum_{k=1}^{\tau} (G_k - {}^bF_k)h_k}, \quad (13)$$

in which $N_1 = N_{t=0}$.

It can be shown by elementary calculus that, as long as G_k and ${}^bF_k$ are strictly functions of the conditions at the start of each step, the limit as $h_k \to 0$ ($\tau \to \infty$) of equation (13) is equation (10)—the true solution. Moreover, the accuracy of equation (13) increases monotonically with τ (see Figure 4). Certainly, a number of other standard formulas (Simpson's rule, finite difference, etc.) can be used to approximate the solution to equation (8), but the recursion described by equation (11) is probably the most familiar to fisheries scientists and has the advantage of providing exact solutions when G_k and ${}^bF_k$ are known constants.

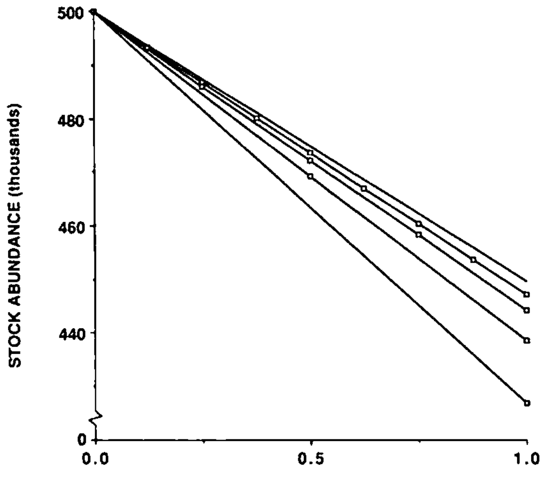

FIGURE 4.—Increasing accuracy of the piecewise exponential approximation routine with decreasing step size (h) for a 1-year simulation. The top curve represents the true solution. The remaining curves, in descending order, represent the solutions when the year is divided into eight, four, two, and one time steps, respectively. (The models used to generate this figure are those discussed in the example section.)

In the last two paragraphs, we introduced the general numerical scheme, but did not completely specify the proper choice of ${}^bF_k$. Equation (12) defines ${}^bF_k$ in terms of three variables: the fishing mortality rate without the bag limit (${}^wF_k$), the catch without the bag limit (${}^wC_k$), and the catch with the bag limit (${}^bC_k$). The variable ${}^wF_k$ (and G_k) is an investigator-supplied function of the conditions (stock abundance) at the beginning of the kth time step. The catch without the bag limit can be approximated by the familiar catch equation

$${}^wC_k \approx N_k \frac{{}^wF_k}{({}^wF_k - G_k)}(1 - e^{(G_k - {}^wF_k)h_k}), \quad (14)$$

which is derived from equation (11) and, therefore, is exact when ${}^wF_k$ and G_k are constants. The catch with the bag limit can be obtained by distorting the c/f distribution. If the bag limit censors the c/f distribution (equation 5), then

$${}^bC_k \approx S_k\left(\sum_{x<b} xP[x \mid N_k] + b\sum_{x\geq b} P[x \mid N_k]\right); \quad (15)$$

S_k is the number of trips made during the kth time step. The $P[x \mid N_k]$ values are the probabilities of catching x fish without the bag limit; thus, they must be expressed as functions of the moments without the bag limit. The mean catch per trip without the bag limit, for example, is obtained from the catch equation

$$^{w}\lambda_k \approx \frac{^{w}C_k}{S_k}.$$

When the moments of $P[x \mid N_k]$, such as the mean (above), are expressed as functions of $^{w}C_k$, the limit as $h_k \rightarrow 0$ of equation (15) is equation (5). On an intuitive level, the combination of equations (14) and (15) can be thought of as reflecting the c/f distributions for the lower limb of Figure 2. The less one strays from the starting point (the smaller the step size) where the stock abundance with the bag limit is the same as the stock abundance without the bag limit (N_k), the closer will be the two respective c/f distributions and, thus, the accuracy of the approximation.

The Bag Limit Algorithm Applied to a Simple Example

In the preceding section, we presented a general algorithm for simulating the effect of a bag limit on catch and stock abundance. In this section, we examine the special case in which the abundance of a cohort declines exponentially (without the bag limit) and catch per fisher-trip is distributed according to the negative binomial. This example serves primarily as a pedagogical tool to help explain the algorithm, but may prove useful as a first-order model of some real scenarios.

Consider the familiar linear differential model of Beverton and Holt (1957):

$$\frac{dN_t}{dt} = -(M + F)N_t.$$

If the fishing mortality rate (F) and natural mortality rate (M) coefficients were both constant, the stock abundance at the end of the time interval $[0, T]$ would be $N_0e^{-(M+F)T}$. With a bag limit, however, the fishing mortality rate coefficient is necessarily a function of N_t, even if the fishing mortality rate without the bag limit is not. If equation (5) is a reasonable model for the catch with the bag limit, then the stock abundance with the bag limit follows from equation (9), in which $G_0 \equiv -M$. As discussed earlier, the exact solution to equation (9) is unknown, but good approximations can be obtained if the interval $[0, T]$ is divided into many (τ number of) small time steps (length $h = T/\tau$), and equation (11) is used to approximate the solution for each step:

$$N_T \approx N_1e^{-\left(MT + h\sum_{k=1}^{\tau} {^{b}F_k}\right)}, \tag{16}$$

where

$$^{b}F_k \approx \frac{S_k\left(\sum_{x \le b} xP[x \mid N_k] + b \sum_{x > b} P[x \mid N_k]\right)}{^{w}C_k}\, {^{w}F_k},$$

and $^{w}C_k$ is given by equation (14) with $G_k = -M$.

All that is left to do now is to close the problem by specifying $P[x \mid N_k]$. Suppose that the negative binomial is a good model for the c/f distribution:

$$P[x \mid N_k] = \frac{(K_k + x - 1)!}{x!(K_k - 1)!}\left(\frac{m_k}{m_k + K_k}\right)^x \cdot \left(1 + \frac{m_k}{K_k}\right)^{-K_k}.$$

From the moment equations, we know that the parameter m_k is equivalent to the mean catch per trip without the bag limit ($^{w}\lambda_k$), and if the variance in catch among trips follows equation (7),

$$K_k = \frac{^{w}\lambda_k}{\alpha(^{w}\lambda_k)^{\beta-1} - 1}.$$

As mentioned earlier, the accuracy of the algorithm increases with the number of time steps τ. The question is how large should τ be in order to minimize the error? Choosing too small a value for τ will likely lead to a large discretization error, but choosing too large a value for τ can lead to a large roundoff error or excessive use of computer time. The behavior of the error for the case at hand is examined in Appendix 1. The analysis indicates that the algorithm converges at a linear rate (doubling τ halves the error) and suggests an optimum τ of about 2,048 intervals/year for the 32-bit VAX system at the University of Miami. On large mainframe computers this optimum is practical; the execution time is minimal (seconds) even for long simulations with multiple cohorts. On the other hand, the same simulations run on an Apple IIe microcomputer took several hours each. Fortunately, for τ greater than 16 steps/year, the error after 1 year of simulation generally will not exceed 5% of the true value of the catch with the bag limit; thus, weekly or biweekly time steps should be sufficient for short (1–2-year) projections.

Discussion

The basic principle behind the bag limit algorithm is simple: given a starting stock abundance, the c/f distribution for a brief time step is simulated by use of catch statistics obtained from a stock abundance model *without* the bag limit. The simulated c/f distribution is then distorted in some manner to give the catch *with* the bag limit, which in turn is used to compute the stock abundance at the end of the time step (start of the next step). We have attempted to present the mathematical elements of the algorithm in as general a form as possible, that is, without specifying the models for stock abundance and the c/f distribution (except in the example). However, we have limited our discussion of the bag limit's effect on the c/f distribution to censoring (equations 4, 5, and 15). The validity of the censoring approach depends on the soundness of assumptions (5), (6), and (7) (we have relaxed assumption 4 by updating stock abundance after very small increments in time). In this section, we examine these assumptions in more detail, and where appropriate, suggest alternative models. As will become apparent, assumptions (4)–(7) are not mutually exclusive. Moreover, the effect of violating one assumption may mitigate the effect of violating another.

Assumption (5): Fisher Behavior

It is convenient to break assumption (5) into four parts: (a) fishers on a party-trip do not pool their catches, (b) fishers obey the bag limit, (c) the number of fish that are released or discarded does not change with the bag limit, and (d) the effort directed towards the target species does not change.

(a) Pooling.—One way fishers in a party can soften the effect of a bag limit is by misrepresenting their excess catch as part of the catch of fishers who have not met their limit. The operational limit becomes the party limit, i.e., the number of fishers in the party (y) multiplied by the number of fish allowed by the daily bag limit (b). Accordingly, assumption (5) should be modified to read "When regulated by a bag limit, individual *parties* cease fishing as soon as the *party* limit is reached. . . ." The catch with the bag limit for all parties with exactly y fishers, ${}^{b}C_y$, would then be obtained by censoring the distribution of catch per party with y fishers (c/yf) at the party limit:

$$ {}^{b}C_y = S_y\left(\sum_{x \le by} xP[x \mid y] + by \sum_{x > by} P[x \mid y] \right); \quad (17) $$

S_y is the number of parties with exactly y fishers, and $P[x \mid y]$ is the probability that a party with y fishers would catch x fish without the bag limit. The total catch with the bag limit, ${}^{b}C$, is obtained by summing equation (17) over all possible values of y.

Inasmuch as equation (4) is only a special case of equation (17), where $y \equiv 1$, we will adopt the more general notation for all subsequent developments. Equation (17) is incorporated into the example of the preceding section in Appendix 2.

(b) Disobeying the bag limit.—The prevalence of bag violations is obviously a function of the likelihood of detection, the consequences of being detected, and the size of the bag limit. If all fishers (parties) with disobedient inclinations are completely undeterred by the bag limit, then the illegal catch made by parties with y participants (${}^{i}C_y$) is

$$ {}^{i}C_y = P[\text{disobey} \mid y, b]\,({}^{u}C_y - {}^{b}C_y); \quad (18) $$

$P[\text{disobey} \mid y, b]$ is the probability that a party with y members will disobey a bag limit of size b, ${}^{b}C_y$ is the legal catch with the bag limit (from equation 17), and ${}^{u}C_y$ is the catch without the bag limit. When some fishers do not disregard the bag limit entirely, but instead limit their risk of detection by keeping only a few more fish than the bag limit allows (perhaps late in the day), we must consider the probability $P[\xi \mid x, y, b]$ that a disobedient party of y fishers who, unregulated, would have caught x fish, will exceed the party limit by a given number ξ:

$$ {}^{i}C_y = S_y P[\text{disobey} \mid y, b] \cdot \sum_{x > by} \sum_{\xi = 0}^{x - by} \xi P[\xi \mid x, y, b] P[x \mid y]. \quad (19) $$

Equation (19) reduces to equation (18) when $P[\xi = x - by \mid x, y, b] \equiv 1$.

(c) Release mortality and discarding.—High postmortem discards or frequent releases coupled with high mortality of released fish will substantially reduce the effectiveness of a bag limit. Bannerot (1987) approached this problem by assuming that fishers *capture* as many fish as they would have without the bag limit, but *keep* only their legal limit and release the rest, some fraction of which will die:

$$ R_y = P[\text{released fish dies} \mid y]({}^{u}C_y - {}^{b}C_y - {}^{i}C_y). \quad (20) $$

He found, for example, that the size of the bag limit required to achieve a 25% reduction in the private recreational catch of Atlantic king mackerel stocks declined from four fish per angler-day at 0% release mortality to one fish per angler-day at 75% release mortality.

Bannerot's (1987) hypothesis is consistent with the commonly held belief that retaining and consuming fish are relatively unimportant motives for recreational fishers (Fedler 1984). However, numerous published studies on human dimensions of fisheries have indicated that some groups of fishers place much more importance on retaining the catch than others (Fedler and Ditton 1986; Matlock et al. 1988; Peyton and Gigliotti 1989). These fishers may stop fishing after they reach their limit (reducing release mortality), or, if the possibility of catching a larger fish is good, they may continue fishing and discard the small, dead fish as larger fish are caught (increasing discard mortality). There is also the possibility that some fishers, in anticipation of catching larger fish, will release small, live fish before they reach their limit, which reduces the chance that they will meet the limit. The importance of these phenomena will vary from fishery to fishery, and we have found it difficult to establish any useful generalization from the published literature on human dimensions. Case-specific hypotheses could be examined by incorporating an age (length) structure into the models in the bag limit algorithm. We suspect that, in most cases, Bannerot's (1987) model will provide a useful upper bound for the true number of fish that actually die as a result of fishing.

(d) Effort.—The total amount of effort put forth by parties with y participants, f_y, can be expressed as the product of the number of trips (S_y) and the average number of fisher-hours per trip (γ_y). Assumption (5) stipulates that fishers stop fishing as soon as the bag limit is reached, implying that the only effect of a bag limit is to reduce γ. It is possible, however, that the bag limit will also influence the number of trips. If the change in the number of trips is independent of the fishers' ability to catch fish, then the shape of the c/yf distribution will be preserved. If this is the case, the algorithm in Appendix 2 would need no further modification, though we emphasize that the term ${}^{w}F_y$ (the potential fishing mortality rate without the bag limit) implicitly includes S_y and must be updated accordingly. As Bannerot (1987) pointed out, however, the catch distribution will probably shift with changes in the total number of trips, because different classes of fishers will respond to the bag limit differently. Skilled fishers may divert their effort towards other fisheries (increasing the bag limit's effectiveness) or attempt to circumvent the limit by including nonfishers in their party (decreasing the bag limit's effectiveness). Fishers for whom retaining the catch is very important (e.g., sustenance) may make more fishing trips. Again, it is difficult to establish a meaningful generalization, but a practical lower limit for the catch with the bag limit would be obtained by assuming that all of the fishers affected by the bag limit simply drop out of the fishery so that the c/yf distribution is truncated rather than censored:

$$ {}^{b}C_y = {}^{b}S_y \frac{\sum_{x \le by} xP[x \mid y]}{\sum_{x \le by} P[x \mid y]} ; \qquad (21) $$

${}^{b}S_y$ is the number of trips with the bag limit,

$$ {}^{b}S_y = {}^{w}S_y \sum_{x < by} P[x \mid y], $$

and ${}^{w}S_y$ is the number of trips without the bag limit. The upper limit is, trivially, the catch without the bag limit. Most likely, the catch with the bag limit will fall between the two extremes (such as equation 17), but there is an infinite range of possibilities. As Ditton and Fedler (1989) commented, "Research is needed . . . that examines angler responses to regulatory measures that are context specific."

Assumption 6: Competing User Groups

Reducing a given group's ability to catch fish will have the net effect of making a larger fraction of the stock available to competing groups. Bag limit regulations, however, have the additional effect of becoming increasingly restrictive as catchability or stock abundance increases. Hence, the ability of the bag-limit-regulated user-groups to capitalize during times of improved fishing may be severely limited compared to that of the competing user-groups. This effect would be especially critical when the effort of both groups fluctuates with the local catchability or abundance of the stock. There is also the possibility that the competing groups will take advantage of the decrease in effort from the bag-limit-regulated sector and fish the stock harder in anticipation of greater catches. Accordingly, one must consider the dynamics of the competing user-groups to avoid systematically overestimating the catch of the bag-limit-regulated group.

Assumption 7: Multispecies Fisheries

Forecasting the effect of a bag limit on the exploitation of multispecies assemblages is an especially difficult task. It requires not only knowledge of the interdependence among the exploited species, but also an understanding of fisher behavior. If, for example, a bag limit were placed on only one of two exploited species, fishers who normally targeted the regulated species might instead target the second species. The problem is exacerbated when the different species are routinely caught in the same areas with the same gear.

A few special cases may be easily resolved. Consider, for example, an assemblage of species that share the same habitat and are equally vulnerable to the same types of gear, e.g., panfish in a lake or grunts on a patch reef. If this assemblage were managed as a single species by a lump bag limit, then, barring discriminatory behavior of the fishers, all of the species would experience the same proportional reduction in catch or fishing mortality rate. The censoring approach, or some other transformation, could then be applied to the ensemble catch distribution.

The scenario changes drastically if the different species interact or if fishers tend to discard one species in favor of the others. Deriso and Parma (1987) presented a compelling probabilistic model of hook-and-line angling that includes interference competition among three species, and there is no reason it could not be adapted to include bag limits. Otherwise, there is a dearth of information on the subject.

We note that the above discussion applies equally well when the phrase "multiple-species assemblages" is amended to include multiple cohorts within a species. With multiple cohorts, we become concerned with changing size-selectivity patterns of the fishers and cohort-to-cohort interactions.

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Appendix 1: Determination of Optimum Step Size and Convergence Rate

Here we present the results of an a posteriori error analysis of the bag limit algorithm. The crux of such an analysis is that the maximum accuracy is realized when the discretization error (associated with the numerical method) and roundoff error (associated with computer word size) are roughly equal (Howard 1974). Typically, the cumulative discretization error over a fixed period is assumed to have the form $\alpha\tau^{-\theta}$; θ is the "order" of the method, τ is the number of subintervals in each period, and α depends on the equations involved in the calculations. Roundoff error is usually treated as a random variable with expectation $\beta\tau$; β depends on the equations and computer word size. Plots of $\log_e |error|$ versus $\log_e \tau$ should then show a straight line with slope $-\theta$ when discretization error is dominant (small τ) and a straight line with slope one when roundoff error is dominant (large τ). The two lines are connected by a curvilinear region in which the roundoff and discretization errors are roughly equal.

Following Howard (1974), we obtained the standard of reference to calculate the error for our analysis as follows.

(1) The simulation was repeated for a sequence of values of τ (1, 2, 4, 8, 16, . . . , 1,048,576), and the computed value of ${}^bC$ was recorded for each τ.
(2) First-order ($\theta = 1$) Richardson extrapolations were made for each successive pair of computed values of ${}^bC$.
(3) When q successive extrapolations agreed, the $q + 1$ computed values lay along a straight line on the log–log graph, and the average of the extrapolations was taken as the standard of reference.

Table A1.1 gives the results from the analysis when the annual fishing mortality rate = 1.0, annual natural mortality rate = 0.5, initial stock abundance = 25,000, number of trips = 145,585, and the variance in catch among trips was equal to the mean catch among trips multiplied by 20. The standard of reference used to compute the error (7,572.471) was the average of the extrapolations for which discretization error was dominant—the five values flagged by single asterisks. The average of the roundoff extrapolations, flagged by double asterisks, was 7,598.104,

TABLE A1.1.—Results from an a posteriori error analysis of the bag limit algorithm (Appendix 1) in which fishers are treated individually ($y \equiv 1$); the annual fishing mortality rate without the bag limit (${}^{w}F$) = 1.0; natural mortality rate (M) = 0.5; initial stock abundance (N_1) = 25,000; number of trips per year (S) = 145,585; bag limit (b) = 5; and the variance (σ^2) in catch among trips = the mean catch per trip (${}^{w}\lambda$) multiplied by 20. Single asterisks (*) indicate estimates of the true catch, with the bag limit in effect, based on extrapolations to zero error when discretization error is dominant (mean = 7,572.471). Double asterisks (**) are extrapolations obtained when roundoff error is dominant (mean = 7,598.104).

Intervals per year	Catch with bag limit: Calculated	Catch with bag limit: Extrapolated	Approximate error
1	6,143.912		−1,428.559
2	6,733.065		−839.406
4	7,116.579		−455.892
8	7,334.792		−237.679
16	7,451.146		−121.325
32	7,511.200		−61.271
64	7,541.732	7,572.264*	−30.739
128	7,557.072	7,572.412*	−15.399
256	7,564.853	7,572.634*	−7.618
512	7,568.674	7,572.495*	−3.797
1,024	7,570.613	7,572.552*	−1.858
2,048	7,571.895		−0.576
4,096	7,572.219		−0.252
8,192	7,573.012		0.541
16,384	7,572.822		0.351
32,768	7,572.860		0.389
65,536	7,575.149		2.678
131,072	7,563.360	7,586.938**	−9.111
262,144	7,588.251	7,538.469**	15.780
524,288	7,588.853	7,587.650**	16.382
1,048,576	7,498.348	7,679.358**	−74.123

which lends credence to the use of 7,572.471 as the standard. The plot of $\log_e |error|$ versus $\log_e \tau$ is shown in Figure A1.1. The error first decreases with the number of intervals per year, is minimized when τ = 4,096 ($\log_e \tau$ = 8.32), and then increases with increasing τ in the noisy fashion one expects when roundoff errors are important. The two respective trends are linear on the log–log scale and have

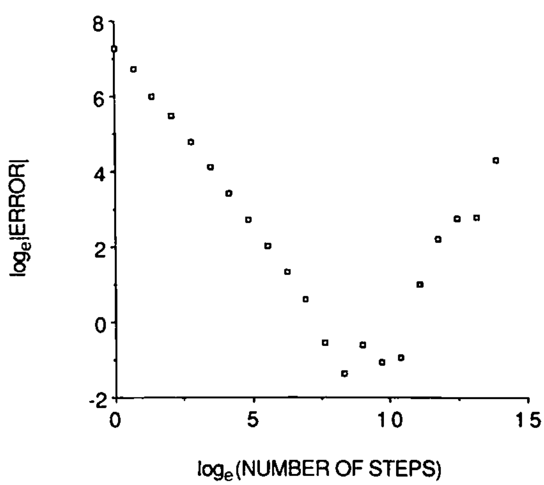

FIGURE A1.1.—Example of a plot of $\log_e |error|$ versus $\log_e \tau$; error = (standard − computed catch) and τ is the number of intervals (steps) into which each year is divided. The specific example shown was generated from the results in Table A.1.

slopes very close to one, demonstrating that the algorithm is stable and converges to the optimum at a linear rate. The small deviations from linearity associated with the very small τ (<32), in which sources of error other than discretization or roundoff may be important, are typical of most numerical methods (B. E. Howard, University of Miami, personal communication).

The above analysis was repeated for combinations of different values for fishing mortality (0.1 and 1.0), natural mortality (0.0 and 0.5), stock abundance (25,000; 500,000; and 10,000,000), and variance: mean relationships ($\sigma^2 = 1.1\lambda$, $\sigma^2 = 20\lambda$). The optima typically were close to $\tau = 2{,}048$ except at the extremes, where the bag limit either had no effect (very small mean catch per day) or an extremely strong effect (very large mean catch, i.e., all trips meet the limit). In those cases, the computed values were essentially the same for all τ less than 4,096. A more interesting result was that the percent difference between the computed catch and the extrapolated (true) catch was always less than 5% for $\tau > 16$; hence, weekly or biweekly time steps are sufficient, which makes the algorithm useful to those without access to high-speed mainframe computers.

Appendix 2: The Step-Wise Procedure

Here we give the essential details of the bag limit algorithm when the negative binomial is used to model the c/yf distribution. Equation (17) is used to model the effect of a bag limit on the c/yf distribution, equation (16) is used to model the change in stock abundance, and the variance in catch among party-trips is a power function of the mean catch per party-trip.

Step 1: compute the total catch without the bag limit of all parties with exactly y members:

$$^{w}C_{yk} \approx N_k \frac{^{w}F_{yk}}{(^{w}F_k + M_k)}\left(1 - e^{-(^{w}F_k + M_k)h}\right),$$

$$^{w}F_k \equiv \sum_{y} {^{w}F_{yk}},$$

and h is the length of the time step in years.

Step 2: generate the theoretical c/yf distribution. Here we use the negative binomial distribution in which the variance follows Taylor's (1961) power law:

$$m_{yk} = \frac{^{w}C_{yk}}{S_{yk}},$$

$$K_{yk} = \frac{m_{yk}}{\alpha(m_{yk})^{\beta-1} - 1},$$

and

$$P[x \mid y, N_k] = \frac{(K_{yk} + x - 1)!}{x!(K_{yk} - 1)!}\left(\frac{m_{yk}}{m_{yk} + K_{yk}}\right)^{x}\left(1 + \frac{m_{yk}}{K_{yk}}\right)^{-K_{yk}}.$$

Step 3: censor the generated c/yf distributions at their respective party limits (b times y) to get the catch with the bag limit for parties with y members:

$$^{b}C_{yk} \approx S_{yk}\left(\sum_{x \leq by} xP[x \mid y, N_k] + by \sum_{x > by} P[x \mid y, N_k]\right).$$

Step 4: compute the fishing mortality rate with the bag limit:

$${}^{b}F_{yk} \approx \frac{{}^{b}C_{yk}}{{}^{w}C_{yk}}\, {}^{w}F_{yk}.$$

Step 5: compute the stock's abundance at the start of the next interval:

$$N_{k+1} \approx N_k e^{-({}^{b}F_k + M_k)h};\ {}^{b}F_k \equiv \sum_{y} {}^{b}F_{yk}.$$

Step 6: set $k = k + 1$ and return to step 1.